Eur. Phys. J. C 51, 731–739 (2007)
DOI 10.1140/epjc/s10052-007-0317-4

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Couplings of $N = 1$ chiral spinor multiplets

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Received: 8 March 2007 /
Published online: 22 May 2007 –

Abstract. We derive the action for $n_L \geq 1$ chiral spinor multiplets coupled to vector and scalar multiplets. We give the component form of the action, which contains gauge invariant mass terms for the antisymmetric tensors in the spinor superfield and additional Green–Schwarz couplings to vector fields. We observe that supersymmetry provides mass terms for the scalars in the spinor multiplet that do not arise from eliminating an auxiliary field. We construct the dual action by explicitly performing the duality transformations in superspace and give its component form.

1 Introduction

Antisymmetric tensor fields B_{mn} naturally appear in the light sector of all string theories. In four space-time dimensions ($D = 4$) massless antisymmetric tensors are dual to scalar fields, while massive tensors are dual to massive vectors. Therefore, in the low energy effective action one has the choice to represent these degrees of freedom in either of two dual representations. Depending on the context one formulation might be more convenient than the other, and for this reason both formulations have generically been developed.

Recently compactification with background fluxes and/or compactifications on generalized geometries have been studied in detail; for recent reviews, see, for example, [1–3]. One novelty in these compactifications is the appearance of massive antisymmetric tensors [4]. As a consequence their description in terms of appropriate supergravities has been worked out [5–15]. In particular in $N = 1$ compactifications of type IIB on Calabi–Yau orientifolds with $O5$- or $O9$-planes a massive antisymmetric tensor appears when both electric and magnetic three-form fluxes are turned on [16]. The corresponding $N = 1$ superspace action was constructed in [10, 11]. Orientifolds of generalized geometries, as discussed for example in [17–20], can feature more than one antisymmetric tensor. Therefore, it is of interest to generalize the analysis of [10, 11] and discuss the couplings of a set of n_L massive antisymmetric tensors to vector and chiral multiplets. This is the purpose of the present paper.

In $N = 1$ supersymmetry the three-form field strength of the antisymmetric tensor is part of a linear multiplet L [21–29]. The antisymmetric tensor itself resides in the chiral spinor multiplet Φ_α. Whenever the antisymmetric tensor is massless the supersymmetric action is described in terms of L only. Any mass term for B_{mn} destroys the two-form gauge invariance. However, with the help of appropriate couplings to vector fields gauge invariance can be restored. The resulting Lagrangian is of the Stückelberg type [30, 31], in which the vector fields provide the 'longitudinal' degrees of freedom to render B_{mn} massive. Put differently, in a unitary gauge the antisymmetric tensor 'eats' a vector field and becomes massive. A similar mechanism can be employed for n_L antisymmetric tensors as long as enough ($n_V \geq n_L$) vector fields are coupled. Therefore, the first goal of this paper is the derivation of a $N = 1$ superspace action for n_L chiral spinor multiplets $\Phi^I_\alpha, I = 1, \ldots, n_L$, coupled to n_V vector multiplets $V^A, A = 1, \ldots, n_V$. Furthermore, the gauge couplings of the vector multiplets are allowed to depend on n_C chiral multiplets $N^i, i = 1, \ldots, n_C$.

As we already stated, a massless antisymmetric tensor is dual to a scalar, while a massive one is dual to a massive vector. This duality is also manifest at the level of superfields, where a linear multiplet is dual to a chiral multiplet, while a massive spinor multiplet is dual to a massive vector multiplet. Thus our second aim is to construct the dual theory in superspace.

This paper is organized as follows. In Sect. 2 we introduce the notions of the linear and the chiral spinor multiplet. By means of the Stückelberg mechanism we construct the most general gauge invariant action for n_L massive spinor multiplets and give its corresponding component form. We discuss the resulting scalar potential, which has not the standard $N = 1$ form due to a contribution from the chiral spinor multiplet. In Sect. 3 we perform the duality

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transformations and rewrite the action in terms of $n_V - n_L$ massless and n_L massive vector multiplets. Finally, in the appendix we present the supersymmetry transformations of the chiral spinor multiplet and give a modification of these transformations that preserves the WZ gauge. This allows us to discuss the order parameters for supersymmetry breaking.

2 Spinor superfields coupled to vector and chiral multiplets

In $N=1$ supersymmetry an antisymmetric tensor B_{mn} is part of a chiral spinor superfield Φ_α, while its three-form field strength H_{mnp} resides in a linear multiplet L. In this section, we consider a set of n_L linear multiplets L^I and the corresponding n_L chiral spinor multiplets Φ^I_α, $I=1,\dots,n_L$. We review some of their properties and construct a gauge invariant action.

The linear multiplet is a real superfield, defined by the constraint [22]

$$D^2 L^I = \bar{D}^2 L^I = 0\,, \tag{1}$$

where D_α is the superspace covariant derivative.[1] The θ expansion of L^I reads

$$\begin{aligned} L^I = {} & C^I + \theta\eta^I + \bar{\theta}\bar{\eta}^I + \frac{1}{2}\theta\sigma^m\bar{\theta}\epsilon_{mnpq}H^{npqI} \\ & - \frac{\mathrm{i}}{2}(\theta\theta)\bar{\theta}\bar{\sigma}^m\partial_m\eta^I - \frac{\mathrm{i}}{2}(\bar{\theta}\bar{\theta})\theta\sigma^m\partial_m\bar{\eta}^I - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box C^I\,. \end{aligned} \tag{2}$$

Here C^I are real scalars, η^I are Weyl fermions and $H^I_{mnp} = \partial_{[m}B^I_{np]}$ are the field strengths of the antisymmetric tensors B^I_{np}.

Each antisymmetric tensor B^I_{mn} is contained in a chiral spinor superfield Φ^I_α defined by [22]

$$L^I = \frac{1}{2}\left(D^\alpha\Phi^I_\alpha + \bar{D}_{\dot{\alpha}}\bar{\Phi}^{\dot{\alpha}I}\right)\,, \qquad \bar{D}_{\dot{\beta}}\Phi^I_\alpha = 0\,. \tag{3}$$

The Φ^I_α enjoy the θ expansion

$$\begin{aligned} \Phi^I_\alpha = {} & \chi^I_\alpha - \theta_\gamma\left(\frac{1}{2}\delta^\gamma_\alpha(C^I + \mathrm{i}E^I) + \frac{1}{4}(\sigma^m\bar{\sigma}^n)_\alpha{}^\gamma B^I_{mn}\right) \\ & + \theta\theta\left(\eta^I_\alpha + \mathrm{i}\sigma_{\alpha\dot{\alpha}}{}^m\partial_m\bar{\chi}^{\dot{\alpha}I}\right)\,, \end{aligned} \tag{4}$$

where χ^I_α are additional Weyl fermions and E^I additional scalars. Due to its definition (3) the L^I are invariant under the gauge transformations

$$\Phi^I_\alpha \longrightarrow \Phi^I_\alpha + \frac{\mathrm{i}}{8}\bar{D}^2 D_\alpha\Lambda^I\,, \tag{5}$$

[1] Throughout the paper we are using the conventions of [32].

where the Λ^I are real superfields. The expressions $\bar{D}^2 D_\alpha\Lambda^I$ are chiral and we therefore can write[2]

$$\begin{aligned} & \frac{\mathrm{i}}{8}\bar{D}^2 D_\alpha\Lambda^I \\ & = -\frac{1}{2}\hat{\lambda}^I_\alpha - \left(\delta^\gamma_\alpha\frac{\mathrm{i}}{2}\hat{D}^I + \frac{1}{4}(\sigma^m\sigma^n)^\gamma_\alpha\left(\partial_m\Lambda^I_n - \partial_n\Lambda^I_m\right)\right)\theta_\gamma \\ & \quad - \frac{\mathrm{i}}{2}\theta\theta m\sigma^m_{\alpha\dot{\alpha}}\partial_m\bar{\hat{\lambda}}^{\dot{\alpha}I}\,. \end{aligned} \tag{6}$$

We immediately see that the fields χ^I_α and E^I defined in the θ-expansion of the superfield Φ^I_α in (4) can be gauged away by $\hat{\lambda}^I_\alpha$ and $\hat{D}^I$ using (5). This leaves only the physical degrees of freedom C^I, B^I_{mn} and η^I in the component expansion of Φ^I_α. Thus in this WZ gauge we have

$$\Phi^I_\alpha = -\theta_\gamma\left(\frac{1}{2}\delta^\gamma_\alpha C^I + \frac{1}{4}(\sigma^m\bar{\sigma}^n)^\gamma_\alpha B^I_{mn}\right) + \theta\theta\eta^I_\alpha\,, \tag{7}$$

and the left-over gauge invariance is the standard two-form gauge invariance

$$B^I_{mn} \to B^I_{mn} + \partial_m\Lambda^I_n - \partial_n\Lambda^I_m\,, \quad C^I \to C^I\,, \quad \eta^I_\alpha \to \eta^I_\alpha\,. \tag{8}$$

The superfields Φ^I_α and L^I can be used to construct a gauge invariant action. The kinetic term is given by

$$\mathcal{L}_{\mathrm{kin}} = -\int \mathrm{d}^2\theta\,\mathrm{d}^2\bar{\theta} K(L^I)\,, \tag{9}$$

where $K(L^I)$ is an arbitrary real function of the L^I. In components, (9) reads

$$\begin{aligned} \mathcal{L}_{\mathrm{kin}} = {} & -\frac{1}{4}K_{IJ}\Big(\left(\partial_m C^J\right)\left(\partial^m C^I\right) \\ & \qquad + \mathrm{i}\left(\eta^I\sigma^m\partial_m\bar{\eta}^J + \bar{\eta}^I\bar{\sigma}^m\partial_m\eta^J\right) + \frac{3}{2}H^{mnpI}H^I_{mnp}\Big) \\ & - \frac{1}{8}K_{IJK}\left(\eta^K\sigma^m\bar{\eta}^I\epsilon_{mnpq}H^{npqJ}\right) \\ & - \frac{1}{4!}K_{IJKL}\left(\frac{3}{2}\eta^I\eta^J\bar{\eta}^K\bar{\eta}^L\right)\,, \end{aligned} \tag{10}$$

where we abbreviated

$$K_{IJ\dots K} \equiv \frac{\partial^n K(C)}{\partial C^I\partial C^J\dots\partial C^K}\,. \tag{11}$$

In addition to the kinetic term we can add a mass term for the B^I_{mn} if we introduce a set of Abelian vector multiplets V^A, $A=1,\dots,n_V$. As we will see they can be used to ensure the gauge invariance (8) and they also provide the necessary degrees of freedom in order to render the

[2] The expansion has the same structure as the field strength of the vector multiplet that we introduce in (12). To avoid confusion with (12) we have hatted the corresponding component fields of (6).

B^I_{mn} massive. Let us denote the field strengths of the vector multiplets by $W^A_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V^A$, with the component expansion

$$W^A_\alpha = -\mathrm{i}\lambda^A_\alpha + \left(\delta^\beta_\alpha D^A - \frac{\mathrm{i}}{2}(\sigma^m\bar{\sigma}^n)_\alpha^{\ \beta} F^A_{mn}\right)\theta_\beta + \theta\theta\sigma^m_{\alpha\dot{\alpha}}\partial_m\bar{\lambda}^{\dot{\alpha}A} \,. \tag{12}$$

Here $F^A_{mn} = \partial_m v^A_n - \partial_n v^A_m$ are the field strengths of n_V $U(1)$ gauge bosons v^A_n.[3] The linear combination

$$2\mathrm{i}m^A_J \Phi^J_\beta - W^A_\beta \tag{13}$$

is gauge invariant under (5), provided we assign the following transformation laws to the V^A:

$$V^A \to V^A + m^A_J \Lambda^J \,, \qquad W^A_\beta \to W^A_\beta - \frac{1}{4} m^A_J \bar{D}^2 D_\beta \Lambda^J \,. \tag{14}$$

In (13) and (14) we have introduced the constant coupling matrix m^A_J, which we demand to be real. The linear combination (13) can be used to build (Lorentz and gauge invariant) mass terms for Φ^J_β. However, this is not the only possible gauge invariant term. Permitting the Lagrangian to be invariant only up to a total derivative we can also add the term $2\int \mathrm{d}^2\theta e_{AI}\Phi^I(W^A - \mathrm{i}m^A_J\Phi^J) + \mathrm{h.c.}$, where e_{AI} is a constant real matrix. Note that in this expression only the symmetric part of the product $e_{AI}m^A_J$ appears. The gauge invariance of this additional term can most easily be seen by first rewriting the term as

$$-2\int \mathrm{d}^4\theta e_{AI} L^I V^A - \left(\mathrm{i}\int \mathrm{d}^2\theta e_{AI} m^A_J \Phi^I \Phi^J + \mathrm{h.c.}\right), \tag{15}$$

where we used $\mathrm{d}^2\theta = -\frac{1}{4}D^2$, (3) and the definition of W_α. Using (5) and (14) we perform the gauge transformations on (15). For the first term we obtain

$$\delta\int \mathrm{d}^4\theta(-2e_{AI}L^I V^A) = -2\int \mathrm{d}^4\theta e_{AI} m^A_J L^I \Lambda^J \tag{16}$$

as $\delta L^I = 0$. Transformation of the second term reads

$$\begin{aligned}
&\delta\int \mathrm{d}^2\theta\big(-\mathrm{i}e_{AI}m^A_J\Phi^I\Phi^J\big) + \mathrm{h.c.}\\
&= -\mathrm{i}\int \mathrm{d}^2\theta e_{AI} m^A_J\left(\frac{\mathrm{i}}{4}\Phi^I\bar{D}^2 D\Lambda^J\right) + \mathrm{h.c.}\\
&= -\int \mathrm{d}^4\theta e_{AI} m^A_J(\Phi^I(D\Lambda^J) + \bar{\Phi}^I(\bar{D}\Lambda^J))\\
&= 2\int \mathrm{d}^4\theta e_{AI} m^A_J \Lambda^J L^I + \text{total derivative}\,,
\end{aligned} \tag{17}$$

where (3) and the chirality of Φ have been used. In (17) again only the symmetric part of $e_{AI}m^A_J$ enters, while the variation in (16) contains also the antisymmetric part.

[3] W^A_α is invariant under the standard U(1) gauge invariance $V^A \to V^A + \Sigma^A + \bar{\Sigma}^A$, where the Σ^A are chiral superfields.

Therefore, gauge invariance of (15) requires one to impose the condition $e_{AI}m^A_J = e_{AJ}m^A_I$.[4] Thus the most general gauge invariant action of n_L massive spinor multiplets coupled to n_V vector multiplets is given by

$$\begin{aligned}
\mathcal{L}_m = \frac{1}{4}\int \mathrm{d}^2\theta\big(&f_{AB}\big(2\mathrm{i}m^A_I\Phi^I - W^A\big)\big(2\mathrm{i}m^B_J\Phi^J - W^B\big)\\
&+ 2e_{AI}\Phi^I\big(W^A - \mathrm{i}m^A_J\Phi^J\big)\big) + \mathrm{h.c.}
\end{aligned} \tag{18}$$

The matrix f_{AB} is the gauge coupling function of the vector multiplets, which can depend holomorphically on additional chiral multiplets, which we denote by $N^i, i = 1,\dots,n_C$.[5] The Lagrangian (18) is our first result, which coincides with the Lagrangian of [10,11] in the limit of one linear multiplet, and which was also given previously in [12].

In components the Lagrangian (18) reads

$$\begin{aligned}
\mathcal{L}_m = &-\frac{1}{4}\mathrm{Re}\, f_{AB}\check{F}^A_{mn}\check{F}^{Bmn} + \frac{1}{8}\mathrm{Im}\, f_{AB}\varepsilon^{klmn}\check{F}^A_{kl}\check{F}^B_{mn}\\
&- \frac{1}{16}\epsilon^{klmn}e_{AI}B^I_{kl}\big(\check{F}^A_{mn} + F^A_{mn}\big) + \frac{1}{2}\mathrm{Re}\, f_{AB}D^A D^B\\
&- \frac{1}{2}\big(e_{AI} + 2\,\mathrm{Im}\, f_{AB}m^B_I\big)C^I D^A\\
&- \frac{1}{2}\mathrm{Re}\, f_{AB}m^A_I m^B_J C^I C^J - \frac{\mathrm{i}}{2}f_{AB}\lambda^A\sigma^m\partial_m\bar{\lambda}^B\\
&- \frac{\mathrm{i}}{2}\bar{f}_{AB}\bar{\lambda}^A\bar{\sigma}^m\partial_m\lambda^B - \frac{1}{2}\big(\mathrm{i}e_{AI} + 2f_{AB}m^B_I\big)\eta^I\lambda^A\\
&- \frac{1}{2}\big(-\mathrm{i}e_{AI} + 2\bar{f}_{AB}m^B_I\big)\bar{\eta}^I\bar{\lambda}^A\\
&- \frac{1}{2\sqrt{2}}\partial_i f_{AB}\big(m^A_I C^I - \mathrm{i}D^A\big)\chi^i\lambda^B\\
&- \frac{1}{2\sqrt{2}}\partial_{\bar{i}}\bar{f}_{AB}\big(m^A_I C^I + \mathrm{i}D^A\big)\bar{\chi}^{\bar{i}}\bar{\lambda}^B\\
&- \frac{1}{2\sqrt{2}}\partial_i f_{AB}\check{F}^A_{mn}\chi^i\sigma^{mn}\lambda^B\\
&- \frac{1}{2\sqrt{2}}\partial_{\bar{i}}\bar{f}_{AB}\check{F}^A_{mn}\bar{\chi}^{\bar{i}}\bar{\sigma}^{mn}\bar{\lambda}^B - \frac{1}{4}F^i\partial_i f_{AB}\lambda^A\lambda^B\\
&- \frac{1}{4}\bar{F}^{\bar{i}}\partial_{\bar{i}}\bar{f}_{AB}\bar{\lambda}^A\bar{\lambda}^B + \frac{1}{8}\chi^i\chi^l\partial_i\partial_l f_{AB}\lambda^A\lambda^B\\
&+ \frac{1}{8}\bar{\chi}^{\bar{i}}\bar{\chi}^{\bar{l}}\partial_{\bar{i}}\partial_{\bar{l}}\bar{f}_{AB}\bar{\lambda}^A\bar{\lambda}^B \,,
\end{aligned} \tag{19}$$

where we defined

$$\check{F}^A_{mn} \equiv F^A_{mn} - m^A_I B^I_{mn} \tag{20}$$

and used as the component expansion of N^i

$$\begin{aligned}
N^i &= A^i + \sqrt{2}\theta\chi^i + \theta\theta F^i\,,\\
f_{AB}(N) &= f_{AB}(A) + \sqrt{2}\theta\chi^i\partial_i f_{AB}(A)\\
&\quad + \theta\theta\left(F^i\partial_i f_{AB}(A) - \frac{1}{2}\chi^i\chi^j\partial_i\partial_j f_{AB}(A)\right).
\end{aligned} \tag{21}$$

[4] We thank U. Theis for discussions on this point.

[5] Of course, we also need to add kinetic terms for the N^i, but since they play no role here we omit them in the following.

(We abbreviate $\partial_i \equiv \frac{\partial}{\partial A^i}$.) The auxiliary fields D^A may be eliminated by their equations of motion

$$D^A = \frac{1}{2}(\operatorname{Re} f)^{-1AB}\Big(\big(e_{BI} + 2\operatorname{Im} f_{BC} m_I^C\big)C^I - \frac{\mathrm{i}}{\sqrt{2}}\big(\partial_i f_{BC}\chi^i\lambda^C - \partial_{\bar{i}}\bar{f}_{BC}\bar{\chi}^{\bar{i}}\bar{\lambda}^C\big)\Big) . \qquad (22)$$

Inserting (22) into (19) we obtain

$$\begin{aligned}
\mathcal{L}_m = &-\frac{1}{4}\operatorname{Re} f_{AB}\check{F}^A_{mn}\check{F}^{Bmn} + \frac{1}{8}\operatorname{Im} f_{AB}\varepsilon^{klmn}\check{F}^A_{kl}\check{F}^B_{mn} \\
&- \frac{1}{16}\epsilon^{klmn}e_{AI}B^I_{kl}\big(\check{F}^A_{mn} + F^A_{mn}\big) - V \\
&- \frac{1}{2}\big(\mathrm{i}e_{AI} + 2f_{AB}m_I^B\big)\eta^I\lambda^A \\
&- \frac{1}{2}\big(-\mathrm{i}e_{AI} + 2\bar{f}_{AB}m_I^B\big)\bar{\eta}^I\bar{\lambda}^A - \frac{\mathrm{i}}{2}f_{AB}\lambda^A\sigma^k\partial_k\bar{\lambda}^B \\
&- \frac{\mathrm{i}}{2}\bar{f}_{AB}\bar{\lambda}^A\bar{\sigma}^k\partial_k\lambda^B - \frac{1}{2\sqrt{2}}\partial_k f_{AB}m_I^A C^I\chi^k\lambda^B \\
&- \frac{1}{2\sqrt{2}}\partial_{\bar{k}}\bar{f}_{AB}m_I^A C^I\bar{\chi}^{\bar{k}}\bar{\lambda}^B \\
&+ \frac{\mathrm{i}}{4\sqrt{2}}(\operatorname{Re} f)^{-1AB} \\
&\times\big(\partial_l f_{AG}\chi^l\lambda^G - \partial_{\bar{l}}\bar{f}_{AG}\bar{\chi}^{\bar{l}}\bar{\lambda}^G\big)\big(e_{BI} + 2\operatorname{Im} f_{BC}m_I^C\big)C^I \\
&+ \frac{1}{16}(\operatorname{Re} f)^{-1AB} \\
&\times\partial_k f_{BC}\big(\partial_l f_{AG}\chi^l\lambda^G - \partial_{\bar{l}}\bar{f}_{AG}\bar{\chi}^{\bar{l}}\bar{\lambda}^G\big)\chi^k\lambda^C \\
&+ \frac{1}{16}(\operatorname{Re} f)^{-1AB} \\
&\times\partial_{\bar{k}}\bar{f}_{BC}\big(\partial_{\bar{l}}\bar{f}_{AG}\bar{\chi}^{\bar{l}}\bar{\lambda}^G - \partial_l f_{AG}\chi^l\lambda^G\big)\bar{\chi}^{\bar{k}}\bar{\lambda}^C \\
&- \frac{1}{2\sqrt{2}}\partial_k f_{AB}\check{F}^A_{mn}\chi^k\sigma^{mn}\lambda^B \\
&- \frac{1}{2\sqrt{2}}\partial_{\bar{k}}\bar{f}_{AB}\check{F}^A_{mn}\bar{\chi}^{\bar{k}}\bar{\sigma}^{mn}\bar{\lambda}^B - \frac{1}{4}F^k\partial_k f_{AB}\lambda^A\lambda^B \\
&- \frac{1}{4}\bar{F}^{\bar{k}}\partial_{\bar{k}}\bar{f}_{AB}\bar{\lambda}^A\bar{\lambda}^B + \frac{1}{8}\chi^k\chi^l\partial_k\partial_l f_{AB}\lambda^A\lambda^B \\
&+ \frac{1}{8}\bar{\chi}^{\bar{k}}\bar{\chi}^{\bar{l}}\partial_{\bar{k}}\partial_{\bar{l}}\bar{f}_{AB}\bar{\lambda}^A\bar{\lambda}^B ,
\end{aligned} \qquad (23)$$

where the scalar potential is given by

$$\begin{aligned}
V = \frac{1}{8}\Big(&(\operatorname{Re} f)^{-1AB} \\
&\times\big(e_{AI} + 2\operatorname{Im} f_{AC}m_I^C\big)\big(e_{BJ} + 2\operatorname{Im} f_{BD}m_J^D\big) \\
&+4\operatorname{Re} f_{AB}m_I^A m_J^B\Big)C^I C^J .
\end{aligned} \qquad (24)$$

In order to make the contribution from the D-terms manifest we can alternatively write the potential as

$$V = \frac{1}{2}\operatorname{Re} f_{AB}D^A D^B + \frac{1}{2}\operatorname{Re} f_{AB}m_I^A m_J^B C^I C^J \qquad (25)$$

for $D^A = \frac{1}{2}(\operatorname{Re} f)^{-1AB}(e_{BI} + 2\operatorname{Im} f_{BC}m_I^C)C^I$. We see that there is a contribution to the mass terms for the scalars C^I that does not arise from eliminating an auxiliary field.

For generic charges e_{BJ}, m_J^D (i.e. these are non-zero) the minimum of V is at $C^I = 0$. This follows from the fact that $\operatorname{Re} f_{AB}$ is the gauge kinetic function and therefore is positive definite. As a consequence, both terms in (24) are manifestly positive.

To close our discussion of the Lagrangian (18) let us explicitly display the mass terms for the B^I_{mn}. Using (20) we can write

$$\begin{aligned}
\mathcal{L}^{B2}_m &= -\frac{1}{4}(M^2)_{IJ}B^{Imn}B^J_{mn} \\
&\quad + \frac{1}{8}\big(M^2_{\mathrm{T}}\big)_{IJ}\varepsilon^{klmn}B^I_{kl}B^J_{mn} , \\
(M^2)_{IJ} &= \operatorname{Re} f_{AB}m_I^A m_J^B , \\
\big(M^2_{\mathrm{T}}\big)_{IJ} &= \operatorname{Im} f_{AB}m_I^A m_J^B + \frac{1}{2}e_{AI}m_J^A .
\end{aligned} \qquad (26)$$

As we see the action contains an ordinary mass term M^2 as well as a topological mass term M^2_{T}. For $m_I^A = 0$ both mass terms vanish, and a massless antisymmetric tensor with a Green–Schwarz coupling of the form $e_{AI}\epsilon^{mnpq}F^A_{mn}B^I_{pq}$ is left.

3 Dual formulation

So far we have discussed the possible couplings of a set of spinor superfields to Abelian vector and chiral multiplets. In components this led to massive antisymmetric tensors possibly with additional Green–Schwarz couplings. It is well known that theories with antisymmetric tensors have an equivalent dual formulation: a massive antisymmetric tensor is dual to a massive vector, while a massless antisymmetric tensor is dual to a scalar. The purpose of this section is to derive the dual of the theories discussed in the previous section. More specifically, we perform a duality transformation in superfields and then expand the dual action in components. As a warm-up we first consider the massless case with non-trivial Green–Schwarz couplings ($m_I^A = 0$, $e_{AI} \neq 0$) and then turn to the general case, in which also $m_I^A \neq 0$.

3.1 Massless tensors with Green–Schwarz couplings

For $m_I^A = 0$ the action given by (18) and (9) can be rewritten as

$$\mathcal{L} = -\int \mathrm{d}^4\theta\big(K(L^I) + e_{AI}L^I V^A\big) + \frac{1}{4}\Big(\int \mathrm{d}^2\theta f_{AB}W^A W^B + \mathrm{h.c.}\Big) , \qquad (27)$$

where we partially integrated using the definition of W^A_α, (3) and $\mathrm{d}^2\bar{\theta} = -\frac{1}{4}\bar{D}^2$. We see that the entire action is expressed in terms of linear multiplets only, and no mass term for the antisymmetric tensors is present. The Lagrangian (27) can be derived from the following first-order

Lagrangian:

$$\mathcal{L}_{\text{first}} = -\int \mathrm{d}^4\theta \big(K(V^{0I}) + e_{AI}V^{0I}V^A + V^{0I}(S_I+\bar{S}_I)\big) + \frac{1}{4}\bigg(\int \mathrm{d}^2\theta f_{AB}W^AW^B + \text{h.c.}\bigg), \tag{28}$$

where the V^{0I} denote n_L real vector (but not linear) superfields, and S_I are n_L chiral superfields. Eliminating the S_I by their equations of motion, we find

$$\bar{D}^2 V^{0I} = D^2 V^{0I} = 0\,, \tag{29}$$

where we used that a chiral S_I can always be written in terms of an unconstrained superfield X_I via $S_I = \bar{D}^2 X_I$. From (29) we learn that V^{0I} is constrained to be a linear superfield and thus can be identified as

$$V^{0I} = L^I\,. \tag{30}$$

Inserted back into (28) using $\int \mathrm{d}^4\theta L^I(S_I+\bar{S}_I)=0$, we finally arrive at (27).

If we eliminate the V^{0I} instead we obtain the dual theory in terms of the chiral multiplets S_I. The equation of motion for V^{0I} reads

$$\partial_{V^{0K}}K(V^{0I}) + e_{AK}V^A + S_K + \bar{S}_K = 0\,. \tag{31}$$

With the help of (31) we can express V^{0K} as a function of $e_{AK}V^A + S_K + \bar{S}_K$ and possibly of the other V^{0I}, $I\neq K$. Let us denote this function by h^K, i.e.

$$V^{0K} \equiv h^K\big(V^{0I}, e_{AK}V^A + S_K + \bar{S}_K\big)\,. \tag{32}$$

The precise relation will of course depend on the particular form of $K(V^{0I})$. We may now rewrite K in terms of h^K and replace it by its Legendre transform $\hat{K}$,

$$-\hat{K}\big(e_{AI}V^A + S_I + \bar{S}_I\big) = K(h^J) + \big(e_{AI}V^A + S_I + \bar{S}_I\big)h^I, \tag{33}$$

which, due to (31), is a function of $e_{AK}V^A + S_K + \bar{S}_K$. Inserted into (28) we finally arrive at

$$\mathcal{L} = \int \mathrm{d}^4\theta \hat{K}\big(e_{AI}V^A + S_I + \bar{S}_I\big) + \frac{1}{4}\bigg(\int \mathrm{d}^2\theta f_{AB}W^AW^B + \text{h.c.}\bigg)\,. \tag{34}$$

$\mathcal{L}$ is the dual Lagrangian of (27), which is expressed in terms of n_V vector multiplets V^A and n_L chiral multiplets S_I.

In the original formulation given in (27), the gauge invariance of the vector multiplets

$$V^A \to V^A + \Sigma^A + \bar{\Sigma}^A, \qquad \bar{D}_{\dot{\alpha}}\Sigma^A = 0 \tag{35}$$

is manifest, since the entire action is expressed in terms of the gauge invariant field strength W^A. In the dual formulation (35) has to be accompanied by a shift of the chiral multiplets,

$$S_I \to S_I - e_{AI}\Sigma^A. \tag{36}$$

We see that the S_I play the role of Goldstone supermultiplets, which are necessary in order to maintain the U(1) gauge invariance. Thus the first term in (34) corresponds to a mass term for the vector fields, while the second term is the standard kinetic term. In order to see this more explicitly let us expand the Lagrangian (34) in components. We take V^A in a Wess–Zumino gauge and expand accordingly

$$\begin{aligned} V^A &= -\theta\sigma^m\bar{\theta}v_m^A + \mathrm{i}\theta\theta\bar{\theta}\bar{\lambda}^A - \mathrm{i}\bar{\theta}\bar{\theta}\theta\lambda^A + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D^A\,,\\ S_I &= \frac{1}{2}E_I + \sqrt{2}\theta\psi_I + \theta\theta F_I\,. \end{aligned} \tag{37}$$

Inserted into (34), we arrive at

$$\begin{aligned} \mathcal{L} = &-\frac{1}{4}\operatorname{Re} f_{AB}F^{Amn}F^B_{mn} + \frac{1}{8}\operatorname{Im} f_{AB}\epsilon^{mnpr}F^A_{mn}F^B_{pr}\\ &+\frac{1}{2}\operatorname{Re} f_{AB}D^AD^B\\ &-\frac{\mathrm{i}}{2}\big(f_{AB}\lambda^A\sigma^m\partial_m\bar{\lambda}^B - \bar{f}_{AB}\partial_m\lambda^A\sigma^m\bar{\lambda}^B\big)\\ &+\frac{\mathrm{i}}{2\sqrt{2}}\partial_i f_{AB}D^A\chi^i\lambda^B - \frac{\mathrm{i}}{2\sqrt{2}}\partial_{\bar{i}}\bar{f}_{AB}D^A\bar{\chi}^{\bar{i}}\bar{\lambda}^B\\ &-\frac{1}{2\sqrt{2}}\partial_i f_{AB}F^A_{mn}\chi^i\sigma^{mn}\lambda^B - \frac{1}{2\sqrt{2}}\partial_{\bar{i}}\bar{f}_{AB}F^A_{mn}\bar{\chi}^{\bar{i}}\bar{\sigma}^{mn}\bar{\lambda}^B\\ &-\frac{1}{4}F^i\partial_i f_{AB}\lambda^A\lambda^B - \frac{1}{4}\bar{F}^{\bar{i}}\partial_{\bar{i}}\bar{f}_{AB}\bar{\lambda}^A\bar{\lambda}^B\\ &+\frac{1}{8}\chi^i\chi^j\partial_i\partial_j f_{AB}\lambda^A\lambda^B + \frac{1}{8}\bar{\chi}^{\bar{i}}\bar{\chi}^{\bar{j}}\partial_{\bar{i}}\partial_{\bar{j}}f_{AB}\bar{\lambda}^A\bar{\lambda}^B\\ &+\frac{1}{2}\hat{K}_I e_{AI}D^A + \frac{1}{4}\hat{K}_{IJKL}\psi^I\psi^J\bar{\psi}^K\bar{\psi}^L\\ &+\hat{K}_{IJ}\bigg\{-\frac{1}{4}\partial^m(\operatorname{Re}E_I)\partial_m(\operatorname{Re}E_J)\\ &-\frac{1}{4}\big(\partial^m(\operatorname{Im}E_I) + e_{AI}v^{Am}\big)\big(\partial^m(\operatorname{Im}E_J) + e_{BJ}v^{Bm}\big)\\ &+\frac{\mathrm{i}}{\sqrt{2}}e_{AI}\big(\psi_J\lambda^A - \bar{\psi}_J\bar{\lambda}^A\big)\\ &-\frac{\mathrm{i}}{2}\big(\psi_J\sigma^m\partial_m\bar{\psi}_I + \bar{\psi}_J\bar{\sigma}^m\partial_m\psi_I\big) + F_I\bar{F}_J\bigg\}\\ &+\frac{1}{2}\hat{K}_{IJK}\big\{-\psi_I\sigma^m\bar{\psi}_J\big(e_{AK}v^{Am} + \partial^m(\operatorname{Im}E_K)\big)\\ &-(\psi_I\psi_J)\bar{F}_K - (\bar{\psi}_I\bar{\psi}_J)F_K\big\}\,, \end{aligned} \tag{38}$$

where we abbreviate $\hat{K}_I = \partial_{\operatorname{Re}E_I}\hat{K}$. As promised, we see that the real scalars $(\operatorname{Im}E_K)$ play the role of n_L Goldstone bosons, which render the linear combinations $e_{AK}v^{Am}$ of the n_L vector fields v^{Am} massive.

Eliminating the auxiliary fields F_I and D^A by their equations of motion, we arrive at the following bosonic action:

$$\begin{aligned} \mathcal{L}_{\mathrm{b}} = &-\frac{1}{4}\operatorname{Re} f_{AB}F^{Amn}F^B_{mn} + \frac{1}{8}\operatorname{Im} f_{AB}\epsilon^{mnpr}F^A_{mn}F^B_{pr}\\ &-\frac{1}{4}\hat{K}_{IJ}\big(\partial^m(\operatorname{Re}E_I)\partial_m(\operatorname{Re}E_J) + e_{AI}e_{BJ}v^A_m v^{Bm}\big)\\ &-V\,, \end{aligned} \tag{39}$$

where we have chosen the unitary gauge and absorbed $\operatorname{Im}E_K$ into a redefinition of v^A_m. The scalar potential is of the standard $N=1$ form and is given by

$$V = \frac{1}{2}\operatorname{Re} f_{AB} D^A D^B = \frac{1}{8}(\operatorname{Re} f)^{-1CD} e_{CI} e_{DJ} \hat{K}_I \hat{K}_J \,. \tag{40}$$

This potential agrees with the one given in (25) for $m_I^A = 0$ if one also identifies $\hat{K}_I = -C^I$. For this substitution also the kinetic terms of the scalars C^I and $(\operatorname{Re} E_I)$ agree. Indeed starting from $-\frac{1}{4}K_{IJ}(\partial_m C^J)(\partial^m C^I)$ and using the above identification, we arrive at $-\frac{1}{4}\hat{K}_{IJ}(\partial^m \operatorname{Re} E_I)(\partial_m \operatorname{Re} E_J)$, taking into account that due to (33) we have $\partial_{\operatorname{Re} E_K} K_J = -\delta_K^J$.

3.2 Massive antisymmetric tensors

Let us now turn on the couplings m_I^A and repeat the analysis of the previous section. In this case we start from the first-order Lagrangian

$$\mathcal{L}_{\text{first}} = \int \mathrm{d}^4\theta \left\{ \hat{K}(\widetilde{V}^I) - \frac{1}{2}\widetilde{V}^I \left(D^\alpha \Phi_\alpha^I + \bar{D}_{\dot\alpha} \bar{\Phi}^{I\dot\alpha} \right) \right\} + \mathcal{L}_m \,, \tag{41}$$

where $\mathcal{L}_m$ is given in (18). $\hat{K}$ is a real function of the vector multiplets $\tilde{V}^I$, which will turn out to be the Legendre transform of K.

Let us first show that from (41) one can derive the Lagrangian for n_L massive linear multiplets as given by the sum of (18) and (9). To do so, we vary (41) with respect to $\widetilde{V}^J$ and obtain

$$\partial_{\widetilde{V}^J} \hat{K}(\widetilde{V}^I) = \frac{1}{2}\left(D^\alpha \Phi_\alpha^J + \bar{D}_{\dot\alpha} \bar{\Phi}^{J\dot\alpha} \right) = L^J. \tag{42}$$

For appropriate $\hat{K}$, (42) may be solved giving $\widetilde{V}^J$ as a function of L^J and $\widetilde{V}^I$, $I \neq J$. We shall denote this function by $h^K = h^K(L^K, \widetilde{V}^I) \equiv \widetilde{V}^K$. As in the massless case we can express $\hat{K}$ in terms of the h^K. Together with (42), this leads us from (41) to

$$\mathcal{L} = \int \mathrm{d}^4\theta \{ \hat{K}(h^K) - h^I L^I \} + \mathcal{L}_m \,. \tag{43}$$

Due to (42) the expression $-K(L^I) := \hat{K}(h^I) - h^J L^J$ is the Legendre transform of $\hat{K}(h^I)$ with respect to all h^I, i.e., it is a function only of the L^I. Substituting $K(L^I)$ into (43) we have arrived at the Lagrangian for n_L massive linear multiplets as stated above.

Alternatively, we can eliminate the Φ_α^I multiplets, and this yields the desired dual action. To do so, let us first rewrite (41) as

$$\begin{aligned}\mathcal{L}_{\text{first}} = &\int \mathrm{d}^4\theta \hat{K}(\widetilde{V}^I) + \frac{1}{4}\int \mathrm{d}^2\theta f_{AB} W^A W^B + \text{h.c.} \\ &+ \int \mathrm{d}^2\theta \Big\{ \Phi^I \left(\frac{1}{2}\widetilde{W}^I + \frac{1}{2}e_{AI} W^A - \mathrm{i} f_{AB} m_I^B W^A \right) \\ &\qquad - \mu_{IJ}^2 \Phi^I \Phi^J \Big\} + \text{h.c.} \,,\end{aligned} \tag{44}$$

where $\widetilde{W}^J = -\frac{1}{4}\bar{D}^2 D \widetilde{V}^J$ is the field strength of $\widetilde{V}^J$, and we have performed a partial integration. We also introduced the mass matrix

$$\mu_{IJ}^2 := (M^2)_{IJ} + \mathrm{i}(M_T^2)_{IJ} \,, \tag{45}$$

with M^2 and M_{T}^2 being defined in (26). The equation of motion for Φ_α can be obtained from (44) by using again $\Phi_\alpha = \bar{D}^2 X_\alpha$. Demanding μ_{IJ}^2 to be invertible we arrive at

$$\Phi_\alpha^I = \frac{1}{2}(\mu^2)_{IK}^{-1} \left(\frac{1}{2}\widetilde{W}_\alpha^K + \frac{1}{2}e_{AK} W_\alpha^A - \mathrm{i} f_{AB} m_K^B W_\alpha^A \right) . \tag{46}$$

Inserting this back into (41), we obtain

$$\mathcal{L} = \int \mathrm{d}^4\theta \hat{K}(\widetilde{V}^i) + \frac{1}{4}\int \mathrm{d}^2\theta \hat{f}_{\hat{A}\hat{B}} W^{\hat{A}} W^{\hat{B}} + \text{h.c.} , \tag{47}$$

where we have introduced $W^{\hat{A}} := \left(-\frac{1}{2}\widetilde{W}^I, W^A \right)$. So the index $\hat{A}$ takes values $\hat{A} = (I, A) = (1, \ldots, n_L, n_L+1, \ldots, n_L + n_V)$. Furthermore, the $(n_V + n_L) \times (n_V + n_L)$-dimensional gauge coupling matrix $\hat{f}_{\hat{A}\hat{B}}$ is given by

$$\hat{f}_{\hat{A}\hat{B}} = \begin{pmatrix} \hat{f}_{IJ} & \hat{f}_{IB} \\ \hat{f}_{AJ} & \hat{f}_{AB} \end{pmatrix} , \tag{48}$$

where

$$\begin{aligned}&\hat{f}_{IJ} = (\mu^2)_{IJ}^{-1} \,, \qquad \hat{f}_{IA} = (\mu^2)_{IK}^{-1} \left(-\frac{1}{2}e_{AK} + \mathrm{i} f_{AD} m_K^D \right) , \\ &\hat{f}_{AB} = f_{AB} + (\mu^2)_{IJ}^{-1} \left(-\frac{1}{2}e_{AI} + \mathrm{i} f_{AD} m_I^D \right) \\ &\qquad\qquad \times \left(-\frac{1}{2}e_{BJ} + \mathrm{i} f_{BC} m_J^C \right) .\end{aligned} \tag{49}$$

The term $\hat{K}(\widetilde{V}^I)$ in the Lagrangian (47) contains mass terms for n_L vector multiplets. Thus the Lagrangian (47) appears to depend on n_V massless and n_L massive vector multiplets. However, n_L of the original n_V vector fields are now unphysical. This can be seen from the fact that the gauge coupling matrix $\operatorname{Re} \hat{f}_{\hat{A}\hat{B}}$ has n_L zero eigenvalues, while $\operatorname{Im} \hat{f}_{\hat{A}\hat{B}}$ has n_L constant eigenvalues. Or, in other words, n_L of the vector fields only have a topological coupling but no kinetic term. Indeed, using (26) and (45) it is easy to verify that

$$\hat{f}_{IB} m_K^B = \mathrm{i}\delta_{IK} \,, \qquad \hat{f}_{AB} m_K^B = -\frac{\mathrm{i}}{2}e_{AK} \,. \tag{50}$$

This shows that the n_L vectors $(0, \ldots, 0, m_K^B)$ are eigenvectors of $\operatorname{Re} \hat{f}_{\hat{A}\hat{B}}$ with eigenvalue zero.

In order to display the physical components of $\widetilde{V}$ we decompose it into a vector multiplet V^{0I} in the WZ gauge and the real part of a chiral superfield S^I

$$\widetilde{V}^I := V^{0I} + S^I + \bar{S}^I. \tag{51}$$

The component form of (47) can then be obtained by inserting the Wess–Zumino gauge (37) for V^{0I}, while for the chiral multiplets S^I we use

$$S_I = \frac{1}{2}A_I + \sqrt{2}\theta\psi_I + \frac{\mathrm{i}}{2}\theta\sigma^m\bar{\theta}\partial_m A_I + \theta\theta F_I - \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\partial_m\psi_I\sigma^m\bar{\theta} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\frac{1}{2}A_I\,. \tag{52}$$

Inserting this into (47), we arrive at

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\operatorname{Re}\hat{f}_{\hat{A}\hat{B}}F^{\hat{A}mn}F^{\hat{B}}_{mn} + \frac{1}{8}\operatorname{Im}\hat{f}_{\hat{A}\hat{B}}\varepsilon^{klmn}F^{\hat{A}}_{kl}F^{\hat{B}}_{mn}\\ &+\frac{1}{2}\operatorname{Re}\hat{f}_{\hat{A}\hat{B}}D^{\hat{A}}D^{\hat{B}} + \frac{1}{2}\hat{K}_I D^{0I} - \frac{1}{4}\hat{K}_{IJ}v^{0Im}v^{0J}_m\\ &-\frac{1}{4}\hat{K}_{IJ}\partial^m(\operatorname{Re}A_I)\partial_m(\operatorname{Re}A_J) - \hat{K}_{IJ}F_I\bar{F}_J\\ &-\frac{\mathrm{i}}{2}\big(\hat{f}_{\hat{A}\hat{B}}\lambda^{\hat{A}}\sigma^k\partial_k\bar{\lambda}^{\hat{B}} + \bar{\hat{f}}_{\hat{A}\hat{B}}\bar{\lambda}^{\hat{A}}\bar{\sigma}^k\partial_k\lambda^{\hat{B}}\big)\\ &+\frac{1}{2}\hat{K}_{IJ}\big\{\mathrm{i}\sqrt{2}\big(\psi_J\lambda^{0I} - \bar{\psi}_J\bar{\lambda}^{0I}\big)\\ &-\mathrm{i}\big(\psi_J\sigma^m\partial_m\bar{\psi}_I + \bar{\psi}_J\bar{\sigma}^m\partial_m\psi_I\big)\big\}\\ &+\frac{1}{2}\hat{K}_{IJK}\big\{-\psi_I\sigma^m\bar{\psi}_J v^{0K}_m - (\psi_I\psi_J)\bar{F}_K - (\bar{\psi}_I\bar{\psi}_J)F_K\big\}\\ &+\frac{1}{4}\hat{K}_{IJKL}\psi_I\psi_J\bar{\psi}_K\bar{\psi}_L + \dots\,,\end{aligned} \tag{53}$$

where $\hat{K}_I = \partial_{\operatorname{Re}A_I}\hat{K}$ was used, and terms proportional to $\partial_i\hat{f}_{\hat{A}\hat{B}}$ have been neglected.

The next step is to eliminate the auxiliary fields from the Lagrangian. The equations of motion for F_I can be determined in a straightforward manner to be

$$F_I = \frac{1}{2}\hat{K}^{-1}_{IL}\hat{K}_{LJK}\psi_J\psi_K\,. \tag{54}$$

For $D^{\hat{A}}$, however, the situation is more difficult, since some of the vector multiplets are unphysical. In order to remove the unphysical degrees of freedom we fix the gauge invariance of (46).

To this aim let us rewrite (46) in the following way:

$$\begin{aligned}\Phi^I_\beta = &-\frac{1}{2}R_{IK}\bigg\{-\frac{1}{2}\widetilde{W}^K - \left(\frac{1}{2}e_{AK} + \operatorname{Im}f_{AB}m^B_K\right)W^A\\ &+R^{-1}_{KL}I_{LJ}\operatorname{Re}f_{AB}m^B_J W^A\bigg\}\\ &+\frac{\mathrm{i}}{2}I_{IK}\bigg\{\frac{1}{2}\widetilde{W}^K + \left(\frac{1}{2}e_{AK} + \operatorname{Im}f_{AB}m^B_K\right)W^A\\ &-I^{-1}_{KL}R_{LJ}\operatorname{Re}f_{AB}m^B_J W^A\bigg\}\,,\end{aligned} \tag{55}$$

where we divided the coupling matrices of (46) into their real and imaginary parts, and we abbreviated

$$R_{IK} = [\operatorname{Re}((\mu^2)^{-1})]_{IK}$$

and

$$I_{IK} = [\operatorname{Im}((\mu^2)^{-1})]_{IK}\,. \tag{56}$$

Going to the WZ gauge (7) for Φ^I, we see that the θ-component of the imaginary part of the right hand side of (55) has to vanish. This implies

$$\begin{aligned}D^{0K} = &-\big(e_{AK} + 2\operatorname{Im}f_{AB}m^B_K\big)D^A\\ &+2I^{-1}_{KL}R_{LJ}\operatorname{Re}f_{AB}m^B_J D^A + \dots\,,\end{aligned} \tag{57}$$

where we have omitted the fermionic terms. We can now use the constraint (57) to eliminate the D^{0K} from the Lagrangian. Let us concentrate on the bosonic terms, which we read off from (53) to be

$$V = -\frac{1}{2}\operatorname{Re}\hat{f}_{\hat{A}\hat{B}}D^{\hat{A}}D^{\hat{B}} - \frac{1}{2}\hat{K}_I D^{0I}. \tag{58}$$

Inserting (49) and (57) and using $D^{\hat{A}} = \big(-\frac{1}{2}D^{0K}, D^A\big)$, we obtain the following equation of motion for D^A:

$$\begin{aligned}&\big\{R_{IJ}I^{-1}_{JK}I^{-1}_{IN}R_{KL}R_{NS}\operatorname{Re}f_{AC}\operatorname{Re}f_{BD}m^C_L m^D_S\\ &\quad+R_{IJ}\operatorname{Re}f_{AC}\operatorname{Re}f_{BD}m^C_I m^D_J + \operatorname{Re}f_{AB}\big\}D^B\\ &= -\hat{K}_K\bigg\{-\left(\frac{1}{2}e_{AK} + \operatorname{Im}f_{AC}m^C_K\right)\\ &\quad+I^{-1}_{KL}R_{LJ}\operatorname{Re}f_{AC}m^C_J\bigg\}\,.\end{aligned} \tag{59}$$

The inverse of the matrix multiplying D^B is found to be

$$(\operatorname{Re}f)^{-1EA} - R_{TU}m^E_T m^A_U\,, \tag{60}$$

which implies

$$D^E = \hat{K}_K(\operatorname{Re}f)^{-1EA}\left(\frac{1}{2}e_{AK} + \operatorname{Im}f_{AC}m^C_K\right). \tag{61}$$

Inserting (54) and (61) back into (53), we arrive at

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\operatorname{Re}\hat{f}_{\hat{A}\hat{B}}F^{\hat{A}mn}F^{\hat{B}}_{mn} + \frac{1}{8}\operatorname{Im}\hat{f}_{\hat{A}\hat{B}}\varepsilon^{klmn}F^{\hat{A}}_{kl}F^{\hat{B}}_{mn}\\ &-\frac{1}{4}\hat{K}_{IJ}v^{0Im}v^{0J}_m - \frac{1}{4}\hat{K}_{IJ}\partial^m(\operatorname{Re}A_I)\partial_m(\operatorname{Re}A_J)\\ &-\frac{\mathrm{i}}{2}\big(\hat{f}_{\hat{A}\hat{B}}\lambda^{\hat{A}}\sigma^k\partial_k\bar{\lambda}^{\hat{B}} + \bar{\hat{f}}_{\hat{A}\hat{B}}\bar{\lambda}^{\hat{A}}\bar{\sigma}^k\partial_k\lambda^{\hat{B}}\big)\\ &+\frac{1}{2}\hat{K}_{IJ}\big\{\mathrm{i}\sqrt{2}\big(\psi_J\lambda^{0I} - \bar{\psi}_J\bar{\lambda}^{0I}\big)\\ &-\mathrm{i}\big(\psi_J\sigma^m\partial_m\bar{\psi}_I + \bar{\psi}_J\bar{\sigma}^m\partial_m\psi_I\big)\big\}\\ &-\frac{1}{2}\hat{K}_{IJK}\psi_I\sigma^m\bar{\psi}_J v^{0K}_m\\ &+\frac{1}{4}\big(\hat{K}_{IJKL} - \hat{K}_{IJM}\hat{K}^{-1}_{MS}\hat{K}_{KLS}\big)\psi_I\psi_J\bar{\psi}_K\bar{\psi}_L\\ &-V + \dots\,,\end{aligned} \tag{62}$$

where the scalar potential is given by

$$\begin{aligned}V = \frac{1}{8}\big\{&\big(e_{AI} + 2\operatorname{Im}f_{AC}m^C_I\big)\\ &\times\operatorname{Re}f^{-1AB}\big(e_{BJ} + 2\operatorname{Im}f_{BD}m^D_J\big)\\ &+4\operatorname{Re}f_{AB}m^A_I m^B_J\big\}\hat{K}_I\hat{K}_J\,.\end{aligned} \tag{63}$$

This potential indeed coincides with (25) for $C^I = \hat{K}^I$. For this identification also the kinetic terms agree, which simply expresses the fact that K and $\hat{K}$ are related by a Legendre transformation. Thus (62) is the desired dual action of (23).

4 Conclusion

Let us summarize our results. We proposed an $N=1$ superfield action for n_L chiral spinor superfields coupled to n_V vector and n_C chiral multiplets. The component form of this action was given and shown to contain gauge invariant mass terms for n_L antisymmetric tensors. In addition, the action also features Green–Schwarz couplings to the n_V vector fields. Supersymmetry gives a mass to the supersymmetric partners C^I of the antisymmetric tensors with the peculiarity that these mass terms do not arise from eliminating an auxiliary field. Indeed, the supersymmetry transformation laws show that any Lorentz invariant ground state of the spinor superfield preserves supersymmetry. Instead the supersymmetry transformations of the vector multiplets are modified, and a vacuum expectation value of the scalars C^I can break supersymmetry by generating a non-vanishing gaugino transformation.

We also constructed the dual action in terms of n_L massive and $n_V - n_L$ massless vector multiplets by explicitly performing the duality transformations in superspace. We gave the component form of the dual action and showed that the scalar potentials in both formulations coincide.

For one chiral spinor superfield, the action agrees with the action given in [10, 11], which also appeared in Kaluza–Klein reduction of type IIB string theory compactified on Calabi–Yau orientifolds [16].

Acknowledgements. This work is supported by The German Science Foundation (DFG) and the European RTN Program MRTN-CT-2004-503369.

We thank Ferdinand Brennecke, Olaf Hohm, Waldemar Schulgin and Ulrich Theis for useful discussions.

Appendix: Modified SUSY transformations of the chiral spinor superfield

In this appendix we derive the supersymmetry transformation laws of a chiral spinor superfield in the WZ gauge. The main motivation for this exercise is to identify the order parameters for spontaneous supersymmetry breaking. For simplicity, we perform this analysis for a single Φ_α.

The general supersymmetry transformation of Φ_α reads

$$\Phi_\alpha \to \Phi'_\alpha = \Phi_\alpha + \delta_\xi \Phi_\alpha = \Phi_\alpha + (\xi Q + \bar{\xi}\bar{Q})\Phi_\alpha \,, \qquad (A.1)$$

where Q and $\bar{Q}$ are the supersymmetry generators. In terms of the component expansion (4), we have

$$\begin{aligned}
\delta_\xi \chi_\alpha &= -\xi_\gamma \left(\frac{1}{2}\delta^\gamma_\alpha (C + \mathrm{i}E) + \frac{1}{4}(\sigma^m\bar{\sigma}^n)_\alpha^{\ \gamma} B_{mn} \right) , \\
\delta_\xi C &= \xi^\alpha \eta_\alpha + \bar{\xi}_{\dot{\alpha}} \bar{\eta}^{\dot{\alpha}} \,, \\
\delta_\xi E &= -\mathrm{i}\xi^\alpha \eta_\alpha + \mathrm{i}\bar{\xi}_{\dot{\alpha}} \bar{\eta}^{\dot{\alpha}} \\
&\quad + 2\left(\xi^\beta \sigma^m_{\beta\dot{\alpha}} \partial_m \bar{\chi}^{\dot{\alpha}} - \bar{\xi}_{\dot{\alpha}} \bar{\sigma}^{m\dot{\alpha}\beta} \partial_m \chi_\beta \right) , \\
\delta_\xi B^{mn} &= 2\eta^\alpha (\sigma^{mn})_\alpha^{\ \beta} \xi_\beta + 2\bar{\eta}_{\dot{\alpha}} (\bar{\sigma}^{mn})^{\dot{\alpha}}_{\ \dot{\beta}} \bar{\xi}^{\dot{\beta}} \\
&\quad + 2\mathrm{i}\left(\xi^\beta \sigma^m_{\beta\dot{\alpha}} \partial^n \bar{\chi}^{\dot{\alpha}} - \xi^\beta \sigma^n_{\beta\dot{\alpha}} \partial^m \bar{\chi}^{\dot{\alpha}} \right) \\
&\quad + 2\mathrm{i}\left(\bar{\xi}_{\dot{\alpha}} \bar{\sigma}^{m\dot{\alpha}\beta} \partial^n \chi_\beta - \bar{\xi}_{\dot{\alpha}} \bar{\sigma}^{n\dot{\alpha}\beta} \partial^m \chi_\beta \right) , \\
\delta_\xi \eta_\alpha &= \mathrm{i}\sigma^k_{\alpha\dot{\alpha}} \bar{\xi}^{\dot{\alpha}} \partial_k C - \frac{1}{2}\varepsilon^{kmnr} \sigma_{r\alpha\dot{\alpha}} \bar{\xi}^{\dot{\alpha}} \partial_k B_{mn} \,. \qquad (A.2)
\end{aligned}$$

In the WZ gauge we choose $\chi_\alpha = 0$ and $E = 0$, and thus (A.2) becomes

$$\begin{aligned}
\delta_{\xi,\mathrm{WZ}} \chi_\alpha &= -\xi_\gamma \left(\frac{1}{2}\delta^\gamma_\alpha C + \frac{1}{4}(\sigma^m\bar{\sigma}^n)_\alpha^{\ \gamma} B_{mn} \right) , \\
\delta_{\xi,\mathrm{WZ}} C &= \xi\eta + \bar{\xi}\bar{\eta} \,, \\
\delta_{\xi,\mathrm{WZ}} E &= -\mathrm{i}\xi\eta + \mathrm{i}\bar{\xi}\bar{\eta} \,, \\
\delta_{\xi,\mathrm{WZ}} B^{mn} &= 2\eta\sigma^{mn}\xi + 2\bar{\eta}\bar{\sigma}^{mn}\bar{\xi} \,, \\
\delta_{\xi,\mathrm{WZ}} \eta_\alpha &= \mathrm{i}(\sigma^k\bar{\xi})_\alpha \partial_k C - \frac{1}{2}\varepsilon^{kmnr}\left(\sigma_r\bar{\xi}\right)_\alpha \partial_k B_{mn} \,. \qquad (A.3)
\end{aligned}$$

We see that χ_α and E do not transform to zero, and, therefore, one needs a compensating gauge transformation to stay in the WZ gauge. These are given in (5) and (6), and so we are led to choose

$$\begin{aligned}
\lambda^e_\alpha &= -\xi_\gamma \left(\frac{1}{2}\delta^\gamma_\alpha C + \frac{1}{4}(\sigma^m\bar{\sigma}^n)_\alpha^{\ \gamma} B_{mn} \right) , \\
D^e &= \mathrm{i}\xi\eta - \mathrm{i}\bar{\xi}\bar{\eta} \,. \qquad (A.4)
\end{aligned}$$

This ensures $(\delta_{\xi,\mathrm{WZ}} + \delta_{\mathrm{gauge}})\chi = 0 = (\delta_{\xi,\mathrm{WZ}} + \delta_{\mathrm{gauge}})E$ and modifies the transformations of the physical fields according to

$$\begin{aligned}
(\delta_{\xi,\mathrm{WZ}} + \delta_{\mathrm{gauge}}) C &= \xi\eta + \bar{\xi}\bar{\eta} \,, \\
(\delta_{\xi,\mathrm{WZ}} + \delta_{\mathrm{gauge}}) B^{mn} &= 2\eta\sigma^{mn}\xi + 2\bar{\xi}\bar{\sigma}^{nm}\bar{\eta} \\
&\quad + \partial^m \Lambda^{en} - \partial^n \Lambda^{em} \,, \\
(\delta_{\xi,\mathrm{WZ}} + \delta_{\mathrm{gauge}}) \eta_\alpha &= \mathrm{i}(\sigma^k\bar{\xi})_\alpha \partial_k C \\
&\quad - \frac{1}{2}\varepsilon^{krmn}\left(\sigma_r\bar{\xi}\right)_\alpha \partial_k B_{mn} \,. \qquad (A.5)
\end{aligned}$$

We see that in a Lorentz invariant ground state supersymmetry cannot be broken by any of these transformations. However, in a WZ gauge the transformation laws of the charged multiplets also change. For the case at hand these are the transformations of the vector multiplet, which without couplings to a spinor superfield read

$$\begin{aligned}
\delta_\xi F_{mn} &= \mathrm{i}\left[\left(\xi\sigma^n\partial_m\bar{\lambda} + \bar{\xi}\bar{\sigma}^n\partial_m\lambda \right) - \left(\xi\sigma^m\partial_n\bar{\lambda} + \bar{\xi}\bar{\sigma}^m\partial_n\lambda \right) \right] , \\
\delta_\xi \lambda_\alpha &= \mathrm{i}\xi_\alpha D + (\sigma^{mn}\xi)_\alpha F_{mn} \,, \\
\delta_\xi D &= \bar{\xi}\bar{\sigma}^m\partial_m\lambda - \xi\sigma^m\partial_m\bar{\lambda} \,. \qquad (A.6)
\end{aligned}$$

Gauge invariance of the couplings to the spinor superfield forces the gauge fields to transform according to (14).

Thus, with the special choice (A.4) we obtain for the combined supersymmetry and gauge transformations

$$\begin{aligned}(\delta_\xi + \delta_{\text{gauge}})\lambda_\alpha &= -m\xi_\gamma\big(\delta_\alpha^\gamma C + \frac{1}{2}(\sigma^m\bar{\sigma}^n)_\alpha^\gamma B_{mn}\big)\\ &\quad + \mathrm{i}\xi_\alpha D + (\sigma^{mn}\xi)_\alpha F_{mn}\,,\\ (\delta_\xi + \delta_{\text{gauge}})D &= m(\mathrm{i}\xi\eta - \mathrm{i}\bar{\xi}\bar{\eta}) + \bar{\xi}\bar{\sigma}^m\partial_m\lambda - \xi\sigma^m\partial_m\bar{\lambda}\,,\\ (\delta_\xi + \delta_{\text{gauge}})F_{mn} &= \mathrm{i}\big[\big(\xi\sigma^n\partial_m\bar{\lambda} + \bar{\xi}\bar{\sigma}^n\partial_m\lambda\big)\\ &\quad - \big(\xi\sigma^m\partial_n\bar{\lambda} + \bar{\xi}\bar{\sigma}^m\partial_n\lambda\big)\big] + mF^e_{mn}\,.\end{aligned} \tag{A.7}$$

As one can see supersymmetry can be broken in (A.7) if either C or D acquire a vacuum expectation value that is different from zero.

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